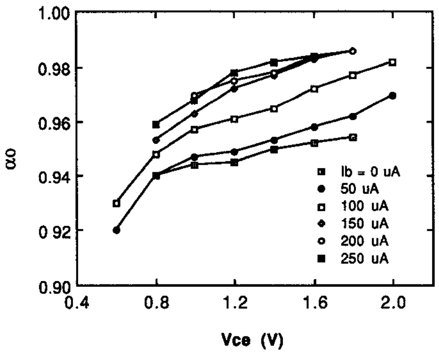

Fig. 5. Plot of the current-transfer ratio (α_o) with variation in the dc bias.

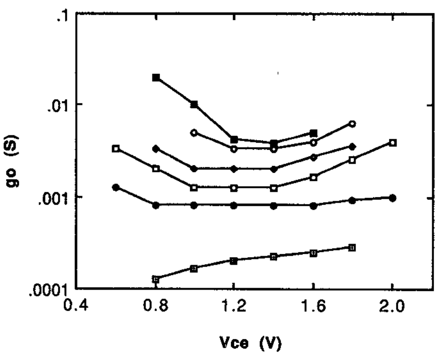

Fig. 6. Plot of the output conductance (g_o) with variation in the dc bias.

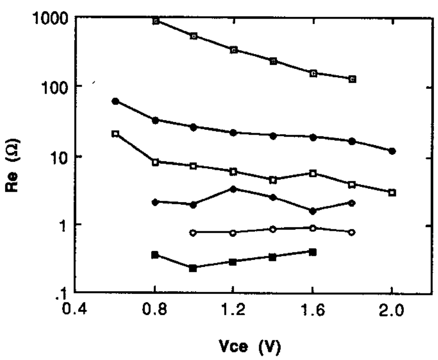

Fig. 7. Plot of the dynamic emitter resistance (R_e) with variation in the dc bias.

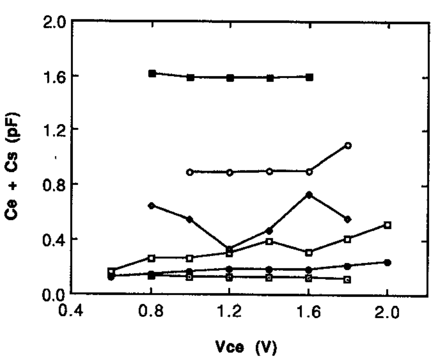

Fig. 8. Plot of the emitter-base diffusion and depletion capacitance ($C_e + C_s$) with variation in the dc bias.

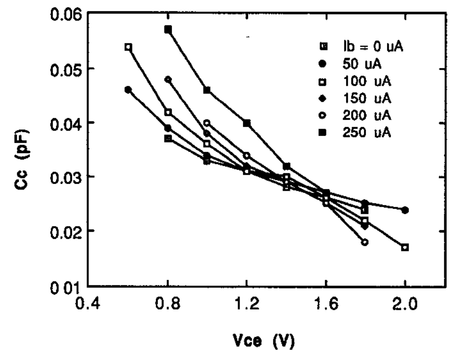

Fig. 9. Plot of the collector-base junction capacitance (C_c) with variation in the dc bias.

saturation, as I_b increases, V_{cb} decreases and since V_{cb} is small, the increase in junction capacitance is significant.

IV. Conclusion

A small signal model has been fitted to S parameter measurements of an inverted InGaAs/InAlAs/InP heterojunction bipolar transistor. The fit was determined over a set of bias values covering the entire useful range of the I-V characteristics. As a result of this measurement and modeling effort, it is clear that consideration of the bias variation of only five intrinsic elements is sufficient to obtain a model valid over a large bias range. Further work on the contribution of each bias dependent element to the overall intermodulation distortion and harmonic distortion is in progress.

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On the Representational Nonuniqueness of Uniform Waveguide Eigenvalue Formulas

P. L. Overfelt

***Abstract*—In the following, we find that for uniform perfectly conducting waveguides characterized by rectilinear boundaries and exact eigenvalue formulas, such formulas are not representationally unique.**

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They are specific examples of general homogeneous polynomials of degree p in q variables, known as q-ary, p-ic forms. Using the concepts of equivalence and congruence, we find that an infinite number of eigenvalue formulas (that are members of an equivalence or congruence class) may be associated with a given waveguide cross section.

I. Introduction

Recently it was pointed out that the eigenvalues (or cutoff wave numbers) of a perfectly conducting uniform waveguide with a 30–60° right triangular cross section can be written in the form [1]

$$k_c^2 = \frac{4\pi^2}{3a'^2}(M^2 + MN + N^2) \tag{1}$$

where M, N are integers ($M, N \in \mathbf{Z}$) and a′ is the side length along the x-axis. A different formula for this waveguide, obtained from the superposition of plane waves technique, has the form

$$k_c^2 = \frac{\pi^2}{3a'^2}(3m^2 + n^2) \tag{2}$$

where $m, n \in \mathbf{Z}$ and the additional restrictions $(m + n)$ and $(m - n)$ are both even [2].

We have observed similar situations for the uniform perfectly conducting equilateral triangular waveguide [2]–[5] and the isosceles right triangular waveguide [2], [6]. From these observations, one might conclude that the nonuniqueness represented by (1) and (2) is a consequence of the nonorthogonal nature of triangular geometries. However, we will show that for uniform perfectly conducting waveguides with boundaries that are convex and rectilinear (i.e., composed of straight lines and a finite number of corners), the exact expression for the eigenvalues is not unique. In particular, we will show that an infinite number of eigenvalue formulas can be associated with a given waveguide cross section and that all of them produce identical sets of eigenvalues. Eigenvalue formulas for the above type of waveguides are specific examples of general q-ary, p-ic forms, which are homogeneous polynomials of degree p in q variables. In particular, all of the known eigenvalue expressions for the above class of waveguides are binary quadratic forms [7], [8].

II. Eigenvalue Formulas and Binary Quadratic Forms

In the following, all waveguides are assumed to be uniform and perfectly conducting with z, t-dependence given by $\exp(-ik_z z + i\omega t)$. Thus we reduce the three-dimensional problem to a two-dimensional one, and the eigenfunctions and eigenvalues are determined by the geometry of the waveguide cross section only. Also all cross section boundaries are assumed to be convex and rectilinear. Within this class of waveguides, some possess closed form expressions for their eigenvalues. Using the simplest example, the rectangular waveguide, we know that this boundary is characterized by the formula

$$k_c^2 = \left(\frac{m\pi}{a'}\right)^2 + \left(\frac{n\pi}{b'}\right)^2, \tag{3}$$

$m, n \in \mathbf{Z}$. Equation (3) is a specific example of a binary quadratic form that can be written generally as (see Appendix for more general definitions)

$$f(m, n) = am^2 + bmn + cn^2 \tag{4}$$

where $m, n \in \mathbf{Z}$. A binary quadratic form is characterized by its coefficients as being real if a, b, c are real (i.e., $a, b, c \in \mathbf{R}$), rational if a, b, c are rational numbers ($a, b, c \in \mathbf{Q}$), or integral if $a, b, c \in \mathbf{Z}$. We refer to m and n as the variables of the form and, in this case, $f(m, n)$ has integral variables.

If we perform a nonsingular linear transformation on (4) using

$$\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} m' \\ n' \end{pmatrix}, \tag{5}$$

then its inverse is

$$\begin{pmatrix} m' \\ n' \end{pmatrix} = \frac{1}{D} \begin{pmatrix} \delta & -\beta \\ -\gamma & \alpha \end{pmatrix} \begin{pmatrix} m \\ n \end{pmatrix} \tag{6}$$

with $D = \alpha\delta - \beta\gamma \neq 0$.

To guarantee that both (m, n) and (m', n') are always integers, we must require that $\alpha, \beta, \gamma, \delta \in \mathbf{Z}$ and restrict D to ± 1. The set of all two-by-two matrices with integer elements and unit determinants is called the unimodular group, and it is a multiplicative infinite group.

If $\alpha, \beta, \gamma, \delta$ are integers but D is not ± 1, then the transformation will be integral in one direction but its inverse may not be integral. Once we have set up (5) and (6) so that both (m, n) and (m', n') are integers, we can always find an $f_2(m', n') = h$ if $f_1(m, n) = h$. Two forms with integer coefficients related by linear transformations with integer elements having this property are called *equivalent* ($f_1 \sim f_2$), and there is no need to consider such forms individually. Applying the concepts of equivalence and congruence to eigenvalue formulas, it is obvious that there can be more than one eigenvalue formula associated with a given waveguide cross section. Thus mode nomenclature is entirely dependent upon the particular form used.

We introduce the following material that will be used to decide when two forms are equivalent (for congruence, see Appendix). We define the matrix of a form (as in (4)) as

$$F = \begin{pmatrix} a & \frac{b}{2} \\ \frac{b}{2} & c \end{pmatrix} \tag{7}$$

and the discriminant of a form as

$$\Delta = 4ac - b^2 = 4|F|. \tag{8}$$

We will assume that the forms of interest are positive definite, meaning that $\Delta > 0$ and $a > 0$. If $\Delta > 0$ and $a > 0$, then $c > 0$ also, and thus the polynomial associated with a given positive definite binary quadratic form is

$$a\xi^2 + b\xi + c \tag{9}$$

and has either complex or pure imaginary zeroes, given by

$$\omega^{\pm} = -\frac{b}{2a} \pm \frac{i\sqrt{\Delta}}{2a}. \tag{10}$$

The zero with positive imaginary part, ω^+, is called the *representative* of $f(m, n)$ in (4). $f(m, n)$ is in "reduced form" [7] if and only if either

$$(1)\ -a \leq b \leq a < c \tag{11a}$$

or

$$(2)\ 0 \leq b \leq a = c. \tag{11b}$$

Using the above concepts and definitions, we can state that if two binary quadratic forms are equivalent, then

1) They must have equal discriminants and

2) When put into reduced form, their reduced forms are identical.

Condition (1) alone is necessary but not sufficient.

Consider the example of a square waveguide with side length π. Its eigenvalue formula is

$$(k_c)^2 = m^2 + n^2; \quad m, n \in \mathbf{Z}. \tag{12}$$

The matrix of this form is

$$F_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad |F_1| = 1 \tag{13}$$

with $\Delta = 4$ and $\omega^+ = i$. Equation (12) satisfies (11b) and thus is already in reduced form. However, the eigenvalues in (12) could be determined from

$$(k_c^2) = k^2 + 2kl + 2l^2 \tag{14}$$

with $k, l \in \mathbf{Z}$, or by

$$(k_c)^2 = i^2 - 2ij + 2j^2, \tag{15}$$

$i, j \in \mathbf{Z}$. The matrix of (14) is

$$F_2 = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}; \quad |F_2| = 1 \tag{16}$$

with $\Delta = 4$, and $\omega^+ = -1 + i$. The matrix of (15) is

$$F_3 = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}; \quad |F_3| = 1 \tag{17}$$

with $\Delta = 4$, and $\omega^+ = 1 + i$.

When (14) and (15) are put into their reduced forms, they are identical to (12). Thus (12), (14), and (15) are equivalent and they are members of an equivalence class. We are now free to consider any member of the class as a means of obtaining the eigenvalues of the square waveguide of side length π. In general, the set of all positive definite binary forms splits up into equivalence classes with any two members of the same class being equivalent and members from different classes being inequivalent. Thus the resulting eigenvalues and the discriminant are invariants that characterize a given equivalence class.

For the example of a square waveguide with side length π, the associated equivalence class of integral binary quadratic forms is infinite. Using (7) and (8), this class can be characterized by

$$G = \begin{pmatrix} 1 & b_o \\ b_o & c \end{pmatrix}, \tag{18}$$

$$|G| = c - b_o^2 = 1 \tag{19}$$

where $b_o = b/2$. Then

$$c = 1 + b_o^2 \tag{20}$$

and this condition can be fulfilled by an infinite number of integer b_o's and c's.

III. Geometric Interpretation

At this point, it is desirable to give a geometric interpretation of the forms in Part II. To do this, we introduce the fundamental integer lattice [9] defined to be a square lattice with unit lengths between intersection points and intersection points at integral values of the orthogonal coordinates. Any binary quadratic form can be written as

$$f(m, n) = am^2 + bmn + cn^2 = K \tag{21}$$

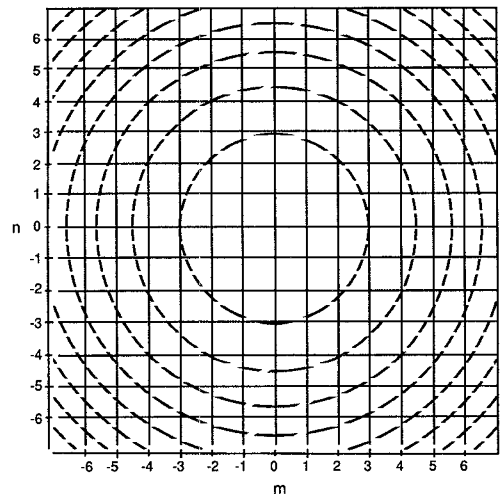

Fig. 1. Contour lines for the eigenvalue formula in (12).

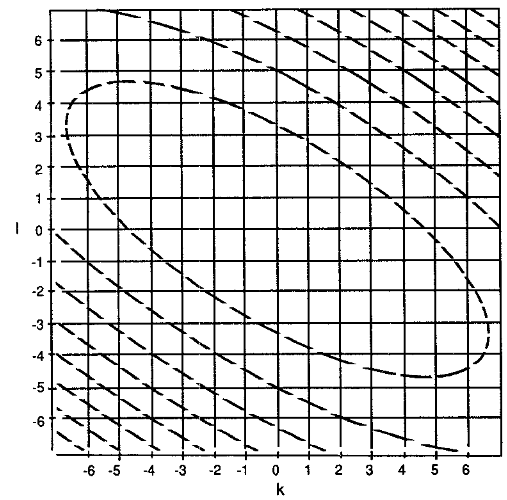

Fig. 2. Contour lines for the eigenvalue formula in (14).

where $m, n, a, b, c, K \in \mathbf{R}$, and K is a constant representing a second-degree curve (conic) with its center at the origin ($m = 0$, $n = 0$). In Figs. 1–3, we show contour lines of (12), (14), and (15), respectively. To obtain continuous contour lines, we have allowed m and n to be real, but for each form the actual eigenvalues occur at those points where the contour lines intersect a lattice point of the fundamental integer lattice. For the previous simple example of the square waveguide, we have $(k_c)^2_{mn} = m^2 + n^2$ whose contour lines (if m and n are allowed to be real) are circles and whose integral values occur when the circles intersect the lattice points. The cutoff wave numbers are given by the radii of those circles that intersect one or more lattice points. There can exist circles of given radii, for which none of the points intersect any lattice point. For this particular example, if $(k_c)^2 \equiv 3 \pmod 4$ [10], then circles with radii equal to $\sqrt{3}$, $\sqrt{7}$, $\sqrt{11}$, etc. never intersect any fundamental integer lattice points; whereas, if $(k_c)^2 \equiv 1 \pmod 4$, then circles with radii equal to $\sqrt{1}$, $\sqrt{5}$, $\sqrt{9}$, $\sqrt{13}$, etc. always intersect at least one integer lattice point.

In general, for (21), if $a = c$ and $b = 0$, the contour lines of the resulting form are circles. If $a \neq c$ and $b = 0$, the contours are ellipses that have semimajor and semiminor axes lined up with the fundamental integer lattice. If $a \neq c$ and $b \neq 0$, the contours are ellipses that are tilted with respect to the lattice. If $b > 0$, the contours are tilted in the counter-clockwise direction (if $a > c$).

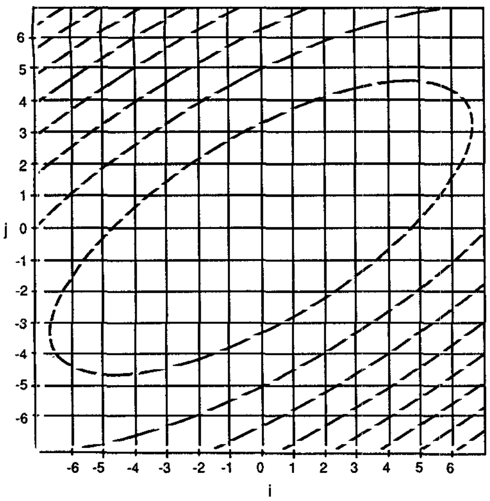

Fig. 3. Contour lines for the eigenvalue formula in (15).

If $b < 0$, the contours are tilted in the clockwise direction (if $a > c$). We see from Figs. 2 and 3 that the contours of (14) and (15) are orthogonal to one another. The tilt angle is found by starting with a form in which the cross-term is present and then rotating so that the cross-term in the new system is eliminated. Thus, the tilt angle of the contour lines with respect to the fundamental integer lattice is given by

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{b}{a-c}\right). \tag{22}$$

IV. 30–60° Right Triangular Waveguide

Taking a', the side length, in (1) and (2) as $a'^2 = 4\pi^2/3$, (1) and (2) become

$$(k_c)^2_{MN} = M^2 + MN + N^2 \tag{23}$$

and

$$(k_c)^2_{mn} = \tfrac{3}{4}m^2 + \tfrac{1}{4}n^2. \tag{24}$$

Obviously these two forms cannot be equivalent since the coefficients of (24) are rational, not integer. However, as stated in the Appendix, if two forms with rational coefficients are related by a nonsingular linear transformation with rational elements, the forms are rationally congruent. In this case, we can transform (24) into (23) using

$$\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} M \\ N \end{pmatrix}. \tag{25}$$

or in matrix form, $\overline{m} = T\overline{M}$, where $|T| = -2$. T is not a member of the unimodular group since its determinant is not ± 1. The inverse of (25) is

$$\begin{pmatrix} M \\ N \end{pmatrix} = \tfrac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} m \\ n \end{pmatrix}. \tag{26}$$

T and T^{-1} form a linear transformation with rational elements. Thus (23) and (24) are rationally congruent. However, in order for $(k_c)^2_{MN}$ and $(k_c)^2_{mn}$ to be identical in value for all $M, N \in \mathbf{Z}$ and all $m, n \in \mathbf{Z}$, something must be added. If we begin by allowing M, N to be any integers, then $(kc)^2_{MN}$ will be integer also. But, if m is an even integer and n is an odd integer or vice versa, then $(kc)^2_{mn}$ will not be integer. This is why the auxiliary restriction $(m + n)$ even, $(m - n)$ even occurs in [2]. This implies that m and n must both be even or they must both be odd. If $m = 2l$ and $n = 2l'$, $l, l' \in \mathbf{Z}$, then

$$\begin{pmatrix} M \\ N \end{pmatrix} = \tfrac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 2l \\ 2l' \end{pmatrix} = \begin{pmatrix} l + l' \\ l - l' \end{pmatrix} \tag{27}$$

and if $m = 2l - 1$, $n = 2l' - 1$, $l, l' \in \mathbf{Z}$, then

$$\begin{pmatrix} M \\ N \end{pmatrix} = \tfrac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 2l - 1 \\ 2l' - 1 \end{pmatrix} = \begin{pmatrix} l + l' - 1 \\ l - l' \end{pmatrix} \tag{28}$$

from which M, N are now guaranteed to be integer once m and n are restricted. Thus (23) and (24) are rationally congruent forms, and the auxiliary condition allows them to give identical sets of eigenvalues because it ensures that the transformation and its inverse are both integral.

V. Conclusion

We have shown that for certain types of uniform waveguides characterized by rectilinear boundaries, their closed form eigenvalue formulas are not representationally unique. Such formulas are specific examples of general homogeneous polynomials of degree p in q variables. All such known expressions turn out to be binary quadratic forms. Using the concepts of equivalence and congruence, we have shown that an infinite number of eigenvalue formulas (that are members of an equivalence or congruence class) can be associated with a given waveguide cross section.

Appendix
Q-ary Quadratic Forms and Congruence

Let

$$s = \sum_{i,j=1}^{q} a_{ij}x_i x_j \quad \text{and} \quad t = \sum_{i,j=1}^{q} b_{ij}y_i y_j \tag{A1}$$

be two q-ary quadratic forms with real variables (x_i, x_j) and (y_i, y_j), and real coefficients a_{ij} and b_{ij}. Assume that $a_{ij} = a_{ji}$, $b_{ij} = b_{ji}$ for all i, j. If a nonsingular linear transformation

$$x_i = \sum_{j=1}^{q} V_{ij}y_j; \qquad i = 1, 2, \cdots, q \tag{A2}$$

with real coefficients, V_{ij}, transforms s into t, then s and t are called *congruent forms* and s is congruent to t ($s \equiv t$). Thus two congruent forms represent the same set of numbers (i.e., take on the same values) as the variables take on all real values. If it is possible to obtain a set of values of y_i making $t = P$, then via (A2) one can obtain a set of values of x_i making $s = P$, and the inverse of (A2) will transform t into s. This correspondence between x_i and y_i is one-to-one. If we are concerned with the values represented by the quadratic forms rather than the forms themselves, then we need to consider only one out of a class of congruent forms.

Equations (A1) and (A2) hold for variables, coefficients, and transformation elements that are all real numbers. However, these forms are called *rationally congruent* when two forms with rational coefficients can be transformed into each other by linear transformations with rational elements. If two forms with integral coefficients can be taken into each other by transformations with integral elements, these forms are called *equivalent*. The variables x_i and y_i can be real, rational, or integral also.

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Effects of Misalignment on Propagation Characteristics of Transmission Lines Printed on Anisotropic Substrates

T. Q. Ho and B. Beker

Abstract—**The spectral-domain method is applied to study the propagation characteristics of grounded transmission lines on biaxial substrates whose axes are misaligned with those of the line. The three structures under investigation are the grounded slotline, microstrip, and the edge coupled line. The formulation derives an expression for the Green's function that is valid for substrates which are simultaneously characterized by both their permittivity and permeability tensors. The off-diagonal elements of the permittivity tensor, present due to the misalignment of the axes, are used to examine the dispersion properties of these transmission lines with numerous case-studies presented for different angles of rotation.**

I. Introduction

Recently, transmission lines on anisotropic materials have become increasingly more attractive in microwave and millimeter-wave integrated circuit applications. Different types of guiding structures such as the microstrip line, coupled line, finline, and slotline on simple anisotropic substrates have been extensively studied by numerous authors in the past. Since the early work on microstrips printed on sapphire substrates, which was presented by Owens *et al.* [1], many other transmission lines on such materials have also been examined in detail. Included among these studies is an open sidewall microstrip which was analyzed by the hybrid mode approach, described by El-Sherbiny [2]. In addition, propagation characteristics of single as well as coupled lines on planar anisotropic layers were also examined via the method of moments, as documented by Alexopoulos *et al.* [3]. Other researchers, such as Nakatani *et al.* [4], have extended the full-wave analysis to study suspended structures, while Yang *et al.* [5] have examined the dispersive properties of a finline. In all of the aforementioned works, however, the anisotropy of a substrate was represented by a diagonal permittivity tensor only.

Up until now, there have been only a few studies dealing with effects of misalignment between the axes of the substrate and those of the waveguide on the dispersive characteristics of transmission lines printed on anisotropic materials. Mathematically, such effects are included by the presence of the off-diagonal elements in the permittivity tensor. Most of the research efforts, thus far, have been primarily focused on open transmission lines, with the most general treatment available in [6]. Therein, Tsalamengas *et al.* have considered an open microstrip line with a substrate which is characterized by generalized $[\epsilon]$ and $[\mu]$ tensors. In their analysis they used a complicated semi-analytical method; however, they did not provide any numerical results for the effects of misalignment on the dispersive properties of the structure. On the other hand, for shielded structures, and specifically for edge coupled lines on a boron nitride, Mostafa *et al.* [7] used a full-wave solution to calculate their dispersion properties for dominant as well as higher order modes.

In this paper, a full-wave analysis applying the spectral-domain technique is used to analyze the effects of misalignment on the propagation characteristics of grounded slotlines, microstrip line, and edge coupled lines printed on biaxial substrates. The material can be characterized simultaneously by both permittivity and permeability tensors, with the rotation of the principal axes restricted to the permittivity tensor alone. Numerical results for various transmission lines are examined in detail with respect to different physical dimensions of the structure, substrate parameters, and angles of rotation of the principal axes of the permittivity tensor.

II. Theory

The specific transmission line structures under consideration are shown in Figs. 1, 2, and 3 along with the coordinate system used to formulate the problem. All metal strips are assumed to be perfectly conducting and infinitely thin. The substrate, whose thickness is h_1, is lossless and is characterized by its permittivity and permeability tensors

$$[\epsilon] = \epsilon_o \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ \epsilon_{yx} & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix} \quad \text{and} \quad [\mu] = \mu_o \begin{bmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix} \tag{1a}$$

with elements of the relative $[\epsilon]$ given by

$$\begin{aligned} \epsilon_{xx} &= \epsilon_2 \sin^2(\theta) + \epsilon_1 \cos^2(\theta), \\ \epsilon_{yy} &= \epsilon_2 \cos^2(\theta) + \epsilon_1 \sin^2(\theta) \\ \epsilon_{zz} &= \epsilon_3, \\ \epsilon_{xy} &= \epsilon_{yx} = (\epsilon_2 - \epsilon_1) \sin(\theta) \cos(\theta), \end{aligned} \tag{1b}$$

where ϵ_o and μ_o are the free-space permittivity and permeability, respectively. The angle θ is the rotation angle of the x and y principal axes of the tensor with respect to the x and y coordinate axes of the structure about the common z-axis.

The vector wave equations for the components of the electric and

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